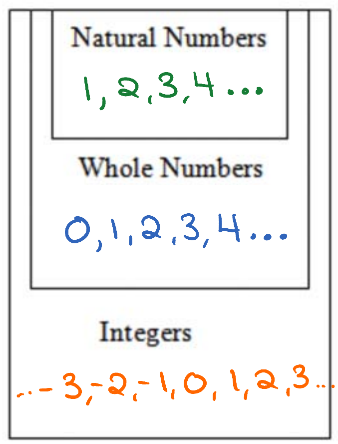
6.1 Graphing Linear Inequalities - Key

Graphing Linear Inequalities in Two Variables [6.1]

Let's start with a little Review. What can you tell me about:



Real Numbers ($\mathbb{R}$)
Natural Numbers (N)
Whole Numbers (W)
Integers (I)

A mathematical inequality must contain one of the following symbols:

dashed/broken line

$<$ $\le$ $>$ $\ge$ $\ne$

solid line

The following are examples of linear inequalities in a single variable :

$4x - 1 > 7$ $1 - 2a \le 5$ etc.

The solution to a single variable inequality can be shown on a number line.

$y = mx + b$ → y-int, → slope

Solid line

Example 1: Graph the solution set for $-3x + 4y \boxed{\le} 12$

Test Point!

Get y alone

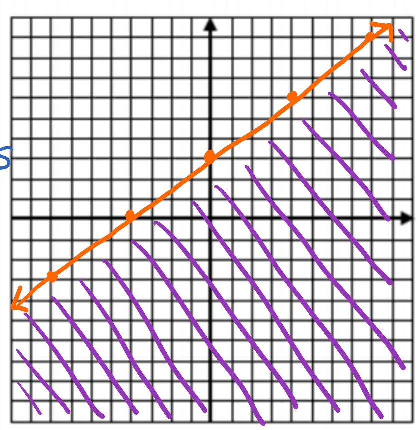
$-3x + 4y \le 12$

$\frac{4y}{4} \le \frac{+3x}{4} + \frac{12}{4}$

$\boxed{y \le \frac{3}{4}x + 3}$

graph $y = \frac{3}{4}x + 3$

Continous graph



When available choose (0,0) as a test point

Test Point (0,0) x, y

Test in ORIGINAL!

$-3x + 4y \le 12$

$-3(0) + 4(0) \le 12$

$0 \le 12$ ✓

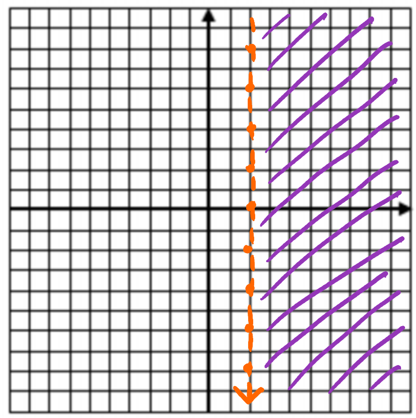
→ Shade area that includes Test Point.

Example 2: Graph the solution set for each linear inequality on a Cartesian plane.

→ dashed line

a) $\{(x, y) \mid x - 2 > 0,\ x \in R, y \in R\}$

↳ Test Point



$x - 2 > 0$

$x > 2$ → Remember: $x = 2$ ↕ $y = 2$ ↔

Test Point (0,0)

$x - 2 > 0$

$0 - 2 > 0$

$-2 > 0$ ✗ False!

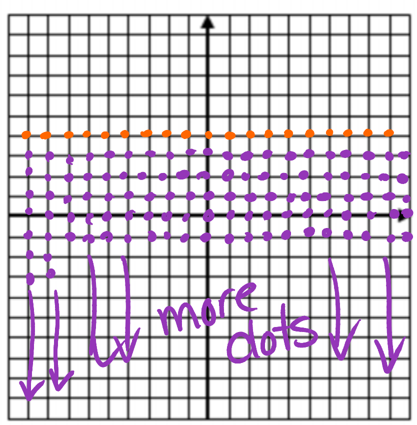
Don't shade area with testpoint

Stipled

↑ line ...

b) $\{(x, y) \mid -3y + 6 \geq -6,\ x \in I, y \in I\}$

↳ Test Point



$-3y + 6 \geq -6$

$\frac{-3y}{-3} \geq \frac{-12}{-3}$

$y \leq +4$

* When we divide or mult. by a negative → we swith the "direction of the sign"

($y = 4$ ↔)

Test Point (0,0)

$-3y + 6 \geq -6$

$-3(0) + 6 \geq -6$

$6 \geq -6$ ✓

(colour in to include Test Point)

Example 3:

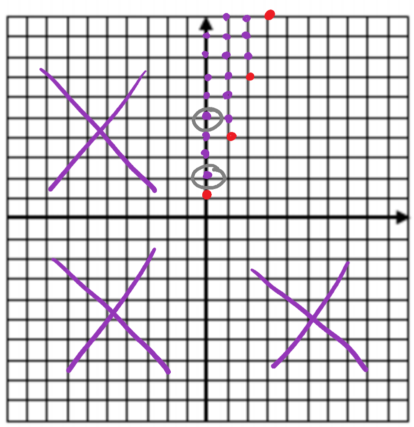
Oliver and Connor are competing in a spelling quiz. Connor gets a point for every word he spells +1 correctly. Oliver is younger than Connor, so he gets 3 points for every word he spells +3 correctly, plus one +1 bonus point. What combinations of correctly spelled words for Oliver and for Connor scoring more points than Oliver? Choose two combinations that make sense and explain your choices.

Let Connor = y Let Oliver = x

$3x+1$

$y > 3x+1$, $x \in W$, $y \in W$

$\frac{Rise}{Run} = \frac{3}{1}$ y-int



Whole #s 0, 1, 2, 3...

Test Point (0,0)

$y > 3x+1$

$0 > 3(0)+1$

$0 > 1$ x False

So "dot" the area that does not include (0,0).

To Answer Question, choose values inside your shaded area.

(0, 2) (0, 5)

x y

Check:

$y > 3x+1$

$2 > 3(0)+1$ ✓

$y > 3x+1$

$5 > 3(0)+1$ ✓

Sooo...

If Connor scores 2 and Oliver 0

OR

If Connor scores 5 and Oliver 0

6.2 Exploring Graphs of Systems - Key

Exploring Graphs of Systems of Linear Inequalities [6.2]

↳ 2 equations

Your solution is where they cross or overlap.

Example 1:

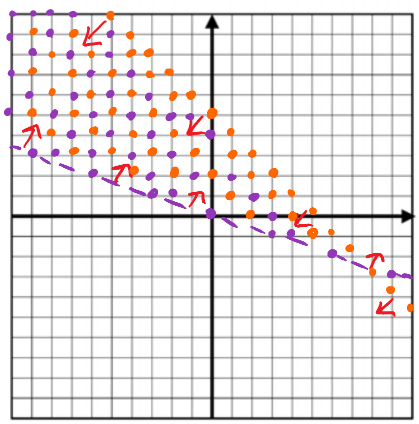
Graph this system of linear inequalities. Justify your representation of the solution set.

solid dots → dots (Integers Only)

① $\{(x,y)|x + y \le 5, \boxed{x \in I, y \in I}\}$

$\{(x,y)|x + 3y > 0, \boxed{x \in I, y \in I}\}$

↳ dashed dots



① $x+y \le 5$ get y alone!

$\boxed{y \le -x+5}$ → start: (0,5) Slope: $-\frac{1}{1}$ $\frac{\text{(rise)}}{\text{(run)}}$

② $x+3y > 0$

$3y > -x + 0$

$\boxed{y > -\frac{1}{3}x}$ → start: (0,0) slope: $-\frac{1}{3}$ $\frac{\text{(rise)}}{\text{(run)}}$

Where do we colour in?
Test points for both to see if our solutions overlap at all.

① Testpoint (0,0)

$x+y \le 5$

$0+0 \le 5$

$0 \le 5$ ✓ true!

② Testpoint (1,1)

$x+3y > 0$

$1+3(1) > 0$

$4 > 0$ ✓ True

* Always test in the original inequality *

$\boxed{\text{Justify}}$ Remember, if they both equal "y" the equal eachother.

$\frac{-3}{1}\left(-\frac{1}{1}x+5\right) = \left(-\frac{1}{3}x\right)\frac{-3}{1}$

· get rid of the fraction.

· get xs alone

$3x - 15 = x$

$2x = 15$

$\boxed{x = 7.5}$

6.3 Graphing to Solve Systems of Linear Inequalities - Key

Graphing to Solve Systems of Linear Inequalities [6.3]

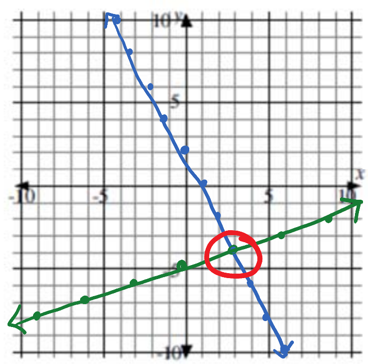
A quick review:

draw straight lines

Consider the system of equations $2x + y = 2$, $x - 3y = 15$.

a) Solve the system of equations graphically by:

- writing both equations in slope y-intercept form
- ~~making a table of values and plotting the points.~~



(3,-4)

① $2x + y = 2$

$\boxed{y = -2x + 2}$

② $x - 3y = 15$

$-3y = -x + 15$

$\boxed{y = \frac{1}{3}x - 5}$

Check (3,-4)

$2x + y = 2$
$2(3) + (-4) = 2$
$6 - 4 = 2$
$2 = 2$ ✓

$x - 3y = 15$
$3 - 3(-4) = 15$
$3 + 12 = 15$
$15 = 15$ ✓

$\boxed{\text{Solution: } (3,-4)}$

Example 1:

Graph the solution set for the following system of inequalities. State two possible solutions from the set. Check your work.

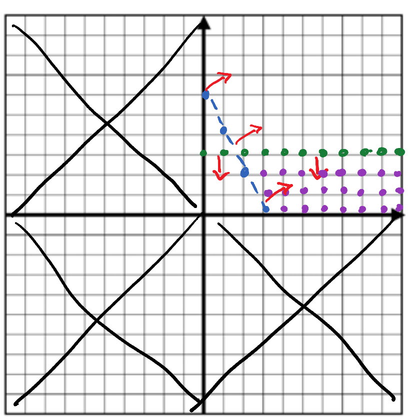
dashed → dots → dots

W = Only Quadrant I

$$\{(x, y) | 2x + y > 6, x \in W, \ y \in W\}$$

dots → dots

$$\{(x, y) | y \leq 3, x \in W, y \in W\}$$



① $2x + y > 6$

$y = -2x + 6$

② $y \leq 3$

Test (0,0)

$2x + y > 6$
$2(0) + 0 > 6$
$0 > 6$ ✗
False

$y \leq 3$
$0 \leq 3$ ✓
True

Solutions
(Any purple dot)
(5,2)
(3,3)

Example 2:

Graph the system and determine a solution.

→ dashed line

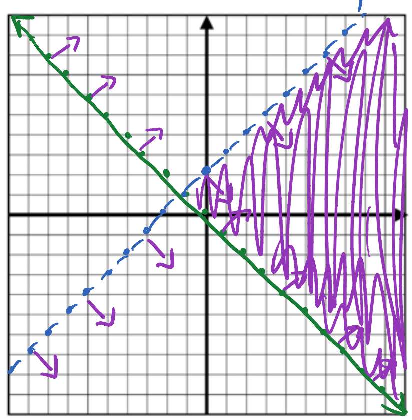
$\{(x, y) | y - x < 2, x \in R, \ y \in R\}$

→ solid line

$\{(x, y) | x + y \geq 0, x \in R, \ y \in R\}$

R = Everything can be coloured. All Quadrants included.

dashed line



① $y - x < 2$

$y < x + 2$

② $x + y \geq 0$

$y \geq -x$ → start: $(0,0)$ slope: $\frac{-1}{1}$

Test $(0,0)$

$y - x < 2$

$0 - 0 < 2$

$0 < 2$ ✓

Test $(1,1)$

$x + y \geq 0$

$1 + 1 \geq 0$

$2 \geq 0$ ✓

$\boxed{\text{Possible Solution: } (5,0)}$

Example 3:

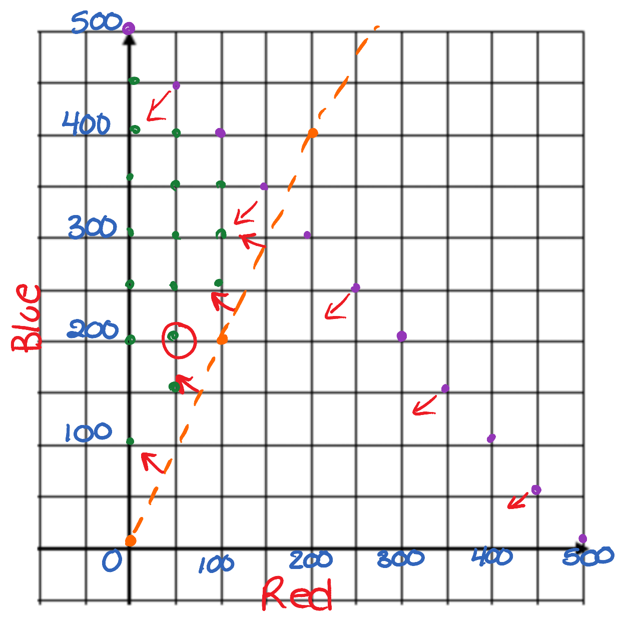
To raise funds to buy new instruments, the band committee has 500 T-shirts to sell. The T-shirts come in red or blue. Based on sales of the same T-shirts at a fundraiser five years ago, the committee expects to sell at least twice as many blue T-shirts as red T-shirts.

a.) Define the variables and restrictions. Write a system of linear inequalities that models the situation.

Red T-shirts : x , $x \in W$ > Can't sell half a shirt...

Blue T-shirts : y , $y \in W$

b.) Graph the system of inequalities.



$x + y \leq 500 \rightarrow y = -x + 500$

$y \geq 2x$

Test (100,100)

$x + y \leq 500$

$100 + 100 \leq 500$

$200 \leq 500$ ✓

Test (100,100)

$y \geq 2x$

$100 \geq 2(100)$

$100 \geq 200$ X

c.) Suggest a combination of T-shirt sales that could be made.

Possibe Solution: (50 Red, 200 Blue)

6.4 Optimization: Creating a Model - Key

Optimization: Creating a Model [6.4]

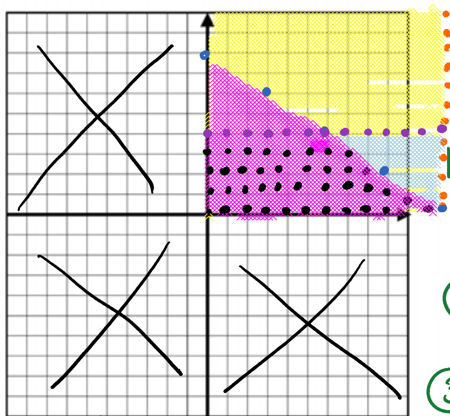
Notice options... cars minivans

Example 1:

Three teams are travelling to a basketball tournament in cars and minivans.

clues
- Each team has no more than 2 coaches and 14 athletes.
- Each car can take 4 team members, and each minivan can take 6 team members.
- No more than 4 minivans and 12 cars are available. → constraints

The school wants to know the combinations or cars and minivans that will require the minimum and maximum number of vehicles. Create a model to represent this situation.



Whole #'s › positives only

... no ½ cars

Options → Let cars = x, $x \in W$
minivans = y, $y \in W$

→ $x = 12$

Inequalities:

① $x \leq 12$ colour left

② $y \leq 4$ colour below

③ $4x + 6y \leq 48$

$6y \leq -4x + 48$

$y \leq -\frac{2}{3}x + 8$ → Test Point (0,0)

$4x + 6y \leq 48$

$4(0) + 6(0) \leq 48$

$0 \leq 48$ ✓ true

each car = 4
each mini = 6
48 = 3 teams ∴
3(2) + 3(14)
↳ TM
↳ coaches

Objective Function
$V = x + y$

* Try "Your Turn" bottom of p. 326 *

Some Necessary Vocab:

- Optimization Problem: A quanity must be maximized or minimized
- Constraint: A limiting condition (represented by linear inequalities)
- Objective function: Equation (=) that represents relationship of 2 variables
- feasible Region: Solution Region for the problem.

Example 2:

A refinery produces oil and gas.

3 • At least 2L of gas is produced for each litre of oil.
- The refinery can produce up to 9 million litres of oil and 6 million litres of gas each day.
- Gasoline is projected to sell of $1.10 per litre. Oil is projected to sell for $1.75 per litre.

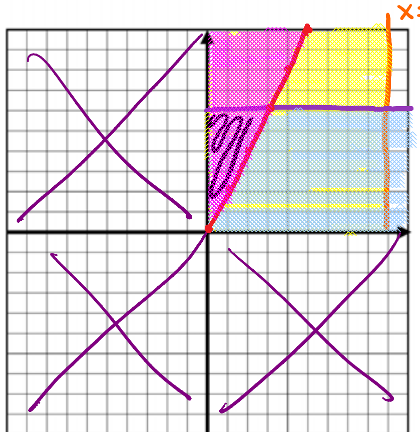
The company needs to determine the daily combination of gas and oil that must be produced to maximize revenue. Create a model to represent the situation.

Optimization

Can you ever really stop the pump @ exactly $20...

Let oil = x , $x \in R$
gas = y , $y \in R$ Restrictions

But any "gas"? Nope
$x \geq 0$, $y \geq 0$



Real gas+oil, Only positives

① $x \leq 9\ 000\ 000$ $\boxed{\text{left}}$
② $y \leq 6\ 000\ 000$ $\boxed{\text{below}}$
③ $y \geq 2x$

Constraints

Test point (3,2)
$2 \geq 2(3)$
$2 \geq 6$ × False

objective function:
$R = 1.10y + 1.75x$

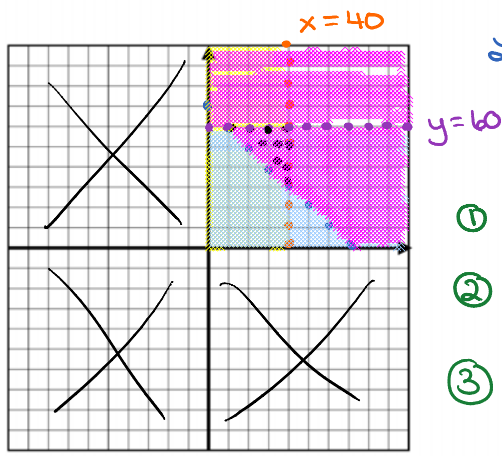
R = Revenue

Example 3: YOUR TURN!

A toy company manufactures two types of toy vehicles: racing cars and sport-utility vehicles.

- Because the supply of materials is limited, no more than 40 racing cars and 60 sport-utility vehicles can be made each day.
- However, the company can make 70 or more vehicles, in total, each day.
- * It costs $8 to make a racing car and $12 to make a sport-utility vehicle.

The company wants to know what combinations will result in the minimum and maximum costs. Create a model to represent the situation.



Real / Whole cars only

Let Racing Cars = x, $x \in W$

SUVs = y, $y \in W$

① $x \leq 40$ $\boxed{\text{left}}$

② $y \leq 60$ $\boxed{\text{below}}$

③ $x + y \geq 70$

$y \geq -x + 70$

↳ Testpoint (1, 1)

$x + y \geq 70$

$1 + 1 \geq 70$

$2 \geq 70$ ✗ Nope!

→ More possibilities but not all are optimizing!

↳ Purpose of 3rd inequality.

Objective Function

$C = 8x + 12y$

Use this to find max + min

To find max + min look at Solutions:

→ (40, 60)

$C = 8x + 12y$

$C = 8(40) + 12(60)$

$C = \$1040$ Max

→ (40, 30)

$C = 8x + 12y$

$C = 8(40) + 12(30)$

$C = \$680$ Min

* Don't believe me → Guess + Check some more *

6.5 Optimization: Exploring Solutions - Key

Optimization: Exploring Solutions [6.5]

Example 1:

Consider the situation:

Restrictions: $x \in R,\ y \in R$ ← solid lines / colour everything

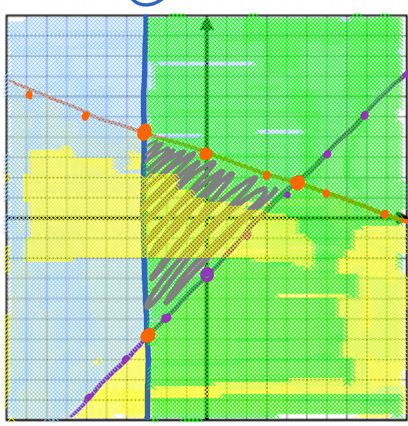
Constraints: $x + 3y \leq 9, x - y \leq 3, x \geq -3$ ← graph inequalities

Use this to find min & max values → Objective function: $P = 2x + y$

a.) Draw a graph to model the situation.

b.) What point in the feasible region would result in the maximum value of the objective function? * look at the points of intersection *

c.) What point in the feasible region would result in the minimum value of the objective function?



① $x + 3y \leq 9$
$3y \leq -x + 9$
$y \leq -\frac{1}{3}x + 3$

Test (0,0)
$x + 3y \leq 9$
$0 + 3(0) \leq 9$
$0 \leq 9$ ✓

② $x - y \leq 3$
$-y \leq -x + 3$
$y \geq x - 3$

Test (0,0)
$x - y \leq 3$
$0 - 0 \leq 3$
$0 \leq 3$ ✓

③ $x \geq -3$
vertical line

Points of Intersection

$(-3, 4)$; $(-3, -6)$; $(4.5, 1.5)$

Objective Function

$P = 2x + y$

$(-3, 4)$
$P = 2x + y$
$P = 2(-3) + 4$
$P = -6 + 4$
$\boxed{P = -2}$

$(-3, -6)$
$P = 2x + y$
$P = 2(-3) + (-6)$
$P = -6 - 6$
$\boxed{P = -12}$
minimum

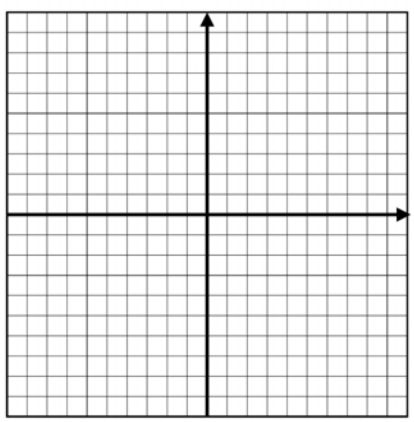
$(4.5, 1.5)$
$P = 2x + y$
$P = 2(4.5) + 1.5$
$P = 9 + 1.5$
$\boxed{P = 10.5}$
maximum

Example 2:

A toy company manufactures two types of toy vehicles: racing cars and sport-utility vehicles.

- Because the supply of materials is limited, no more than 40 racing cars and 60 sport-utility vehicles can be made each day.
- However, the company can make 70 or more vehicles, in total, each day.
- It costs \$8 to make a racing car and \$12 to make a sport-utility vehicle.

The company wants to know what combinations will result in the minimum and maximum costs, and what those costs will be.



6.6 Optimization: Programming - Key

Optimization: Linear Programming [6.6]

Example 1:

The following model represents an optimization problem. Determine the maximum solution.

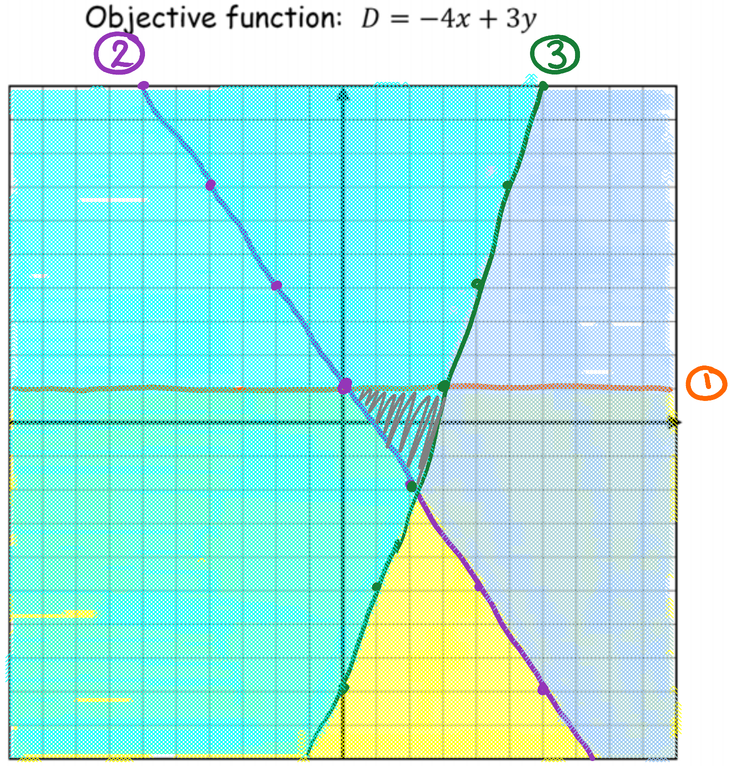
Optimization Model

colour everything all Quadrants solid lines

Restrictions: $x \in R$ and $y \in R$

Constraints: $y \leq 1, 2y \geq -3x + 2, y \geq 3x - 8$

Objective function: $D = -4x + 3y$



① $y \leq 1$ → horizontal line $y = 1$

② $2y \geq -3x + 2$ → Test (0,0)

$y \geq -\frac{3}{2}x + 1$

$2y \geq -3x + 2$

$2(0) \geq -3(0) + 2$

$0 \geq +2$ ✗

False

③ $y \geq 3x - 8$ → Test (0,0)

$y \geq 3x - 8$

$0 \geq 3(0) - 8$

$0 \geq -8$ ✓

True

→ What are the 3 verticies of our feasible region?

(0,1) (3,1) (2,-2)

→ Now put them in the Objective Function

(0,1)

$D = -4x + 3y$

$D = -4(0) + 3(1)$

$\boxed{D = 3}$

maximum

(3,1)

$D = -4x + 3y$

$D = -4(3) + 3(1)$

$D = -12 + 3$

$D = -9$

(2,-2)

$D = -4x + 3y$

$D = -4(2) + 3(-2)$

$D = -8 - 6$

$D = -14$

Example 2:

real items
Whole numbers
only Quadrant I

Larry and Tony are baking cupcakes and banana mini-loaves to sell at a school fundraiser.

less than/equal to

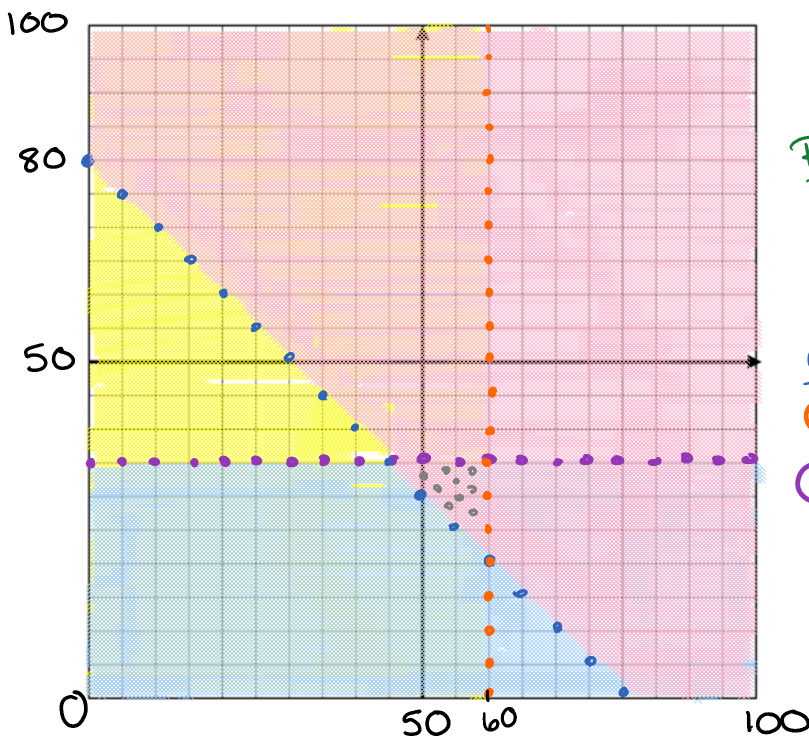
- No more than 60 cupcakes and 35 mini-loaves can be made each day
- Larry and Tony can make more than 80 baked goods, in total, each day.

Constraints

- It costs \$0.50 to make a cupcake and \$0.75 to make a mini-loaf. → objective function

$C = .50x + 0.75y$

They want to know the minimum costs to produce the baked goods.



Options:
Let cupcakes = x
mini-loaves = y

Restrictions

$x \in W$
$y \in W$

Constraints

① $x \leq 60$ colour left

② $y \leq 35$ colour below

$x + y \geq 80$ → Test (0,0)

③ $y \geq -x + 80$

$x + y \geq 80$
$0 \geq 80$
× False

Input verticies into Objective Function > $C = 0.50x + 0.75y$

(45,35)
$C = 0.50x + 0.75y$
$C = 0.5(45) + 0.75(35)$
$C = \$48.75$

(60,35)
$C = 0.50x + 0.75y$
$C = 0.50(60) + 0.75(35)$
$C = \$56.25$

(60,20)
$C = 0.50x + 0.75y$
$C = 0.50(60) + 0.75(20)$
$C = \$45.00$

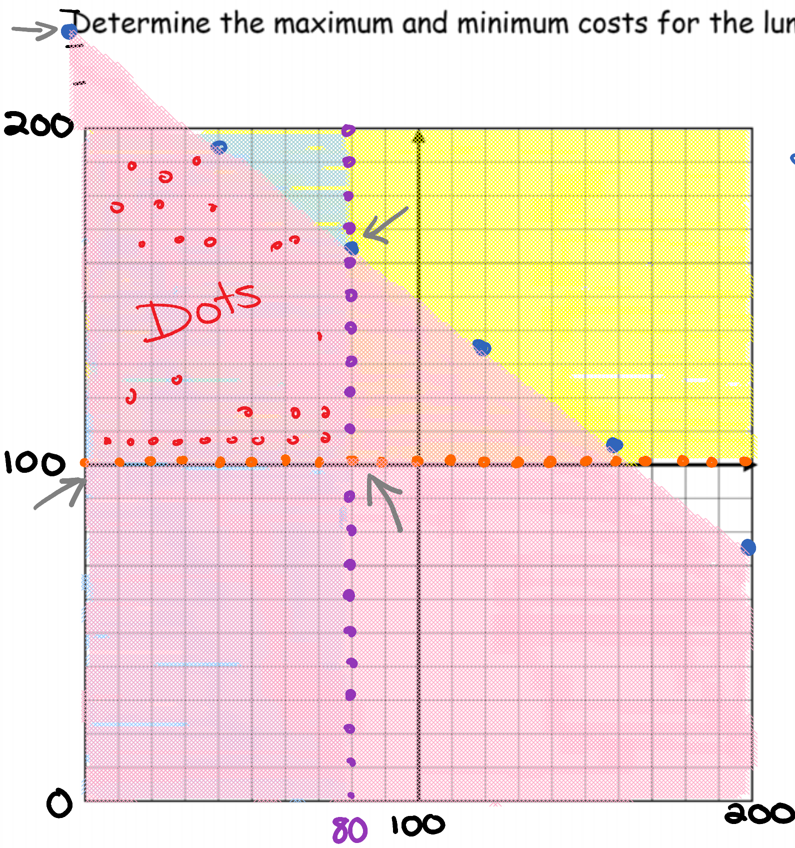
minimum costs

Example 3:

L&G Construction is competing for a contract to build a fence. → Real Fence, Whole Numbers → dots, Only Quadrant I

- The fence will be no longer (less than ≤ 50) than 50yd and will consist of narrow boards that are 6in wide and wide boards that are 8in wide.
- There must be no fewer than 100 wide boards and no more than 80 narrow boards.
- The narrow boards cost \$3.56 each, and the wide boards cost \$4.36 each. → Objective function info

Determine the maximum and minimum costs for the lumber to build the fence.



Options

Let narrow boards $= x$

wide boards $= y$

Restrictions: $x \in W$, $y \in W$

Constraints

① $y \geq 100$ $\boxed{\text{colour above}}$

② $x \leq 80$ $\boxed{\text{colour left}}$

$6x + 8y \leq 50$ yards

inches BUT

so... 1 yard = 36"

50 x 36" = 1800"

$6x + 8y \leq 1800$

$8y \leq -6x + 1800$

③ $y \leq -\frac{3}{4}x + 225$

Test (0,0)

$6x + 8y \leq 1800$

$0 \leq 1800$ ✓ True

Not a triangle > Vertices

Input into $C = 3.56x + 4.36y$

(0, 100) (narrow, wide) → \$436.00 → Minimum Cost

(0, 225) → \$981.00

(80, 100) → \$720.80

(80, 165) → \$1004.20 → Max Cost

(80 ↳ Narrow, 165 ↳ wide)